HBR Classic

Risk analysis in capital investment

David B. Hertz

How can business executives make the best investment decisions? Is there a method of risk analysis to help managers make wise acquisitions, launch new products, modernize the plant, or avoid overcapacity? “Risk Analysis in Capital Investment” takes a look at questions such as these and says “yes”—by measuring the multitude of risks involved in each situation. Mathematical formulas that predict a single rate of return or “best estimate” are not enough. The author’s approach emphasizes the nature and processing of the data used and specific combinations of variables like cash flow, return on investment, and risk to estimate the odds for each potential outcome.

Managers can examine the added information provided in this way to rate more accurately the chances of substantial gain in their ventures. The article, originally presented in 1964, continues to interest HBR readers; the more than 153,000 reprints sold since then testify to the importance of this type of thinking on investment analysis. In a retrospective commentary, the author discusses the now routine use of risk analysis in business and government, emphasizing that the method can—and should—be used in any decision-requiring situations in our uncertain world.

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Of all the decisions that business executives must make, none is more challenging—and none has received more attention—than choosing among alternative capital investment opportunities. What makes this kind of decision so demanding, of course, is not the problem of projecting return on investment under any given set of assumptions. The difficulty is in the assumptions and in their impact. Each assumption involves its own degree—often a high degree—of uncertainty; and, taken together, these combined uncertainties can multiply into a total uncertainty of critical proportions. This is where the element of risk enters, and it is in the evaluation of risk that the executive has been able to get little help from currently available tools and techniques.

There is a way to help the executive sharpen key capital investment decisions by providing him or her with a realistic measurement of the risks involved. Armed with this gauge, which evaluates the risk at each possible level of return, he or she is then in a position to measure more knowledgeably alternative courses of action against corporate objectives.

Need for new concept

The evaluation of a capital investment project starts with the principle that the productivity of capital is measured by the rate of return we expect to receive over some future period. A dollar received next year is worth less to us than a dollar in hand today.
Expenditures three years hence are less costly than expenditures of equal magnitude two years from now. For this reason we cannot calculate the rate of return realistically unless we take into account (a) when the sums involved in an investment are spent and (b) when the returns are received.

Comparing alternative investments is thus complicated by the fact that they usually differ not only in size but also in the length of time over which expenditures will have to be made and benefits returned.

These facts of investment life long ago made apparent the shortcomings of approaches that simply averaged expenditures and benefits, or lumped them, as in the number-of-years-to-pay-out method. These shortcomings stimulated students of decision making to explore more precise methods for determining whether one investment would leave a company better off in the long run than would another course of action.

It is not surprising, then, that much effort has been applied to the development of ways to improve our ability to discriminate among investment alternatives. The focus of all of these investigations has been to sharpen the definition of the value of capital investments to the company. The controversy and furor that once came out in the business press over the most appropriate way of calculating these values have largely been resolved in favor of the discounted cash flow method as a reasonable means of measuring the rate of return that can be expected in the future from an investment made today.

Thus we have methods which are more or less elaborate mathematical formulas for comparing the outcomes of various investments and the combinations of the variables that will affect the investments. As these techniques have progressed, the mathematics involved has become more and more precise, so that we can now calculate discounted returns to a fraction of a percent.

But the sophisticated executive knows that behind these precise calculations are data which are not that precise. At best, the rate-of-return information he is provided with is based on an average of different opinions with varying reliabilities and different ranges of probability. When the expected returns on two investments are close, he is likely to be influenced by intangibles—a precarious pursuit at best. Even when the figures for two investments are quite far apart, and the choice seems clear, there lurk memories of the Edsel and other ill-fated ventures.

In short, the decision maker realizes that there is something more he ought to know, something in addition to the expected rate of return. What is missing has to do with the nature of the data on which the expected rate of return is calculated and with the way those data are processed. It involves uncertainty, with possibilities and probabilities extending across a wide range of rewards and risks. (For a summary of the new approach, see the ruled insert.)

The Achilles heel

The fatal weakness of past approaches thus has nothing to do with the mathematics of rate-of-return calculation. We have pushed along this path so far that the precision of our calculation is, if anything, somewhat illusory. The fact is that, no matter what mathematics is used, each of the variables entering into the calculation of rate of return is subject to a high level of uncertainty.

For example, the useful life of a new piece of capital equipment is rarely known in advance with any degree of certainty. It may be affected by variations in obsolescence or deterioration, and relatively small changes in use life can lead to large changes in return. Yet an expected value for the life of the equipment—based on a great deal of data from which a single best possible forecast has been developed—is entered into the rate-of-return calculation. The same is done for the other factors that have a significant bearing on the decision at hand.

Let us look at how this works out in a simple case—one in which the odds appear to be all in favor of a particular decision. The executives of a food company must decide whether to launch a new packaged cereal. They have come to the conclusion that five factors are the determining variables: advertising and promotion expense, total cereal market, share of market for this product, operating costs, and new capital investment.

On the basis of the "most likely" estimate for each of these variables, the picture looks very bright—a healthy 30% return. This future, however, depends on whether each of these estimates actually comes true. If each of these educated guesses has, for example, a 60% chance of being correct, there is only an 8% chance that all five will be correct (.60 X .60 X .60 X .60 X .60). So the "expected" return actually depends on a rather unlikely coincidence. The decision maker needs to know a great deal more about the other values used to make each of the five estimates and about what he stands to gain or lose from various combinations of these values.

This simple example illustrates that the rate of return actually depends on a specific combination of values of a great many different variables. But
After examining present methods of comparing alternative investments, the author reports on his firm's experience in applying a new approach to the problem. Using this approach, management takes the various levels of possible cash flows, return on investment, and other results of a proposed outlay and gets an estimate of the odds for each potential outcome.

Currently, many facilities decisions are based on discounted cash flow calculations. Management is told, for example, that Investment X has an expected internal rate of return of 9.2%, while for Investment Y a 10.3% return can be expected.

By contrast, the new approach would put in front of the executive a schedule that gives him the most likely return from X, but also tells him that X has 1 chance in 20 of being a total loss, 1 in 10 of earning from 4% to 5%, 2 in 10 of paying from 8% to 10%, and 1 chance in 50 of attaining a 30% rate of return.

From another schedule he learns what the most likely rate of return is from Y, but also that Y has 1 chance in 10 of resulting in a total loss, 1 in 10 of earning from 3% to 5% return, 2 in 10 of paying between 9% and 11%, and 1 chance in 100 of a 30% rate of return.

In this instance, the estimates of the rates of return provided by the two approaches would not be substantially different. However, to the decision maker with the added information, Investment Y no longer looks like the clearly better choice, since with X the chances of substantial gain are higher and the risks of loss lower.

Two things have made this approach appealing to managers who have used it:

1. Certainly in every case it is a more descriptive statement of the two opportunities. And in some cases it might well reverse the decision, in line with particular corporate objectives.
2. This is not a difficult technique to use, since much of the information needed is already available—or readily accessible—and the validity of the principles involved has, for the most part, already been proved in other applications.

The enthusiasm with which management exposed to this approach have received it suggests that it may have wide application. It has particular relevance, for example, in such knotty problems as investments relating to acquisitions or new products and in decisions that might involve excess capacity.

only the expected levels of ranges (worst, average, best; or pessimistic, most likely, optimistic) of these variables are used in formal mathematical ways to provide the figures given to management. Thus predicting a single most likely rate of return gives precise numbers that do not tell the whole story.

The expected rate of return represents only a few points on a continuous curve of possible combinations of future happenings. It is a bit like trying to predict the outcome in a dice game by saying that the most likely outcome is a 7. The description is incomplete because it does not tell us about all the other things that could happen. In Exhibit I, for instance, we see the odds on throws of only two dice having 6 sides. Now suppose that each of eight dice has 100 sides. This is a situation more comparable to business investment, where the company's market share might become any of 100 different sizes and where there are eight factors (pricing, promotion, and so on) that can affect the outcome.
Nor is this the only trouble. Our willingness to bet on a roll of the dice depends not only on the odds but also on the stakes. Since the probability of rolling a 7 is 1 in 6, we might be quite willing to risk a few dollars on that outcome at suitable odds. But would we be equally willing to wager $10,000 or $100,000 at those same odds, or even at better odds? In short, risk is influenced both by the odds on various events occurring and by the magnitude of the rewards or penalties that are involved when they do occur.

To illustrate again, suppose that a company is considering an investment of $1 million. The best estimate of the probable return is $200,000 a year. It could well be that this estimate is the average of three possible returns—a 1-in-3 chance of getting no return at all, a 1-in-3 chance of getting $200,000 per year, a 1-in-3 chance of getting $400,000 per year. Suppose that getting no return at all would put the company out of business. Then, by accepting this proposal, management is taking a 1-in-3 chance of going bankrupt.

If only the best-estimate analysis is used, however, management might go ahead, unaware that it is taking a big chance. If all of the available information were examined, management might prefer an alternative proposal with a smaller, but more certain (that is, less variable) expectation.

Such considerations have led almost all advocates of the use of modern capital-investment-index calculations to plead for a recognition of the elements of uncertainty. Perhaps Ross G. Walker summed up current thinking when he spoke of "the almost impenetrable mists of any forecast."

How can executives penetrate the mists of uncertainty surrounding the choices among alternatives?

Limited improvements

A number of efforts to cope with uncertainty have been successful up to a point, but all seem to fall short of the mark in one way or another:

1. More accurate forecasts—Reducing the error in estimates is a worthy objective. But no matter how many estimates of the future go into a capital investment decision, when all is said and done, the future is still the future. Therefore, however well we forecast, we are still left with the certain knowledge that we cannot eliminate all uncertainty.

2. Empirical adjustments—Adjusting the factors influencing the outcome of a decision is subject to serious difficulties. We would like to adjust them so as to cut down the likelihood that we will make a "bad" investment, but how can we do that without at the same time spoiling our chances to make a "good" one? And in any case, what is the basis for

adjustment? We adjust, not for uncertainty, but for bias.

For example, construction estimates are often exceeded. If a company's history of construction costs is that 90% of its estimates have been exceeded by 15%, then in a capital estimate there is every justification for increasing the value of this factor by 15%. This is a matter of improving the accuracy of the estimate.

But suppose that new-product sales estimates have been exceeded by more than 75% in one-fourth of all historical cases and have not reached 50% of the estimate in one-sixth of all such cases? Penalties for such overestimating are very real, and so management is apt to reduce the sales estimate to "cover" the one case in six—thereby reducing the calculated rate of return. In so doing, it is possibly missing some of its best opportunities.

3. Revising cutoff rates—Selecting higher cutoff rates for protecting against uncertainty is attempting much the same thing. Management would like to have a possibility of return in proportion to the risk it takes. Where there is much uncertainty involved in the various estimates of sales, costs, prices, and so on, a high calculated return from the investment provides some incentive for taking the risk. This is, in fact, a perfectly sound position. The trouble is that the decision maker still needs to know explicitly what risks he is taking—and what the odds are on achieving the expected return.

4. Three-level estimates—A start at spelling out risks is sometimes made by taking the high, medium, and low values of the estimated factors and calculating rates of return based on various combinations of the pessimistic, average, and optimistic estimates. These calculations give a picture of the range of possible results but do not tell the executive whether the pessimistic result is more likely than the optimistic one—or, in fact, whether the average result is much more likely to occur than either of the extremes. So, although this is a step in the right direction, it still does not give a clear enough picture for comparing alternatives.

5. Selected probabilities—Various methods have been used to include the probabilities of specific factors in the return calculation. L. C. Grant discussed a program for forecasting discounted cash flow rates of return where the service life is subject to obsolescence and deterioration. He calculated the odds that the investment will terminate at any time after it is made depending on the probability distribution of the service-life factor. After having calculated these factors for each year through maximum service life, he determined an overall expected rate of return. Edward G. Bennion suggested the use of game theory to take into account alternative market growth rates as they would determine rate of return for various options. He used the estimated probabilities that specific growth rates would occur to develop optimum strategies. Bennion pointed out: "Forecasting can result in a negative contribution to capital budget decisions unless it goes further than merely providing a single most probable prediction... [with] an estimated probability coefficient for the forecast, plus knowledge of the payoffs for the company's alternative investments and calculation of indifference probabilities... the margin of error may be substantially reduced, and the businessman can tell just how far off his forecast may be before it leads him to a wrong decision." 3

Note that both of these methods yield an expected return, each based on only one uncertain input factor—service life in the first case, market growth in the second. Both are helpful, and both tend to improve the clarity with which the executive can view investment alternatives. But neither sharpens up the range of "risk taken" or "return hoped for" sufficiently to help very much in the complex decisions of capital planning.

Sharpening the picture

Since every one of the many factors that enter into the evaluation of a decision is subject to some uncertainty, the executive needs a helpful portrayal of the effects that the uncertainty surrounding each of the significant factors has on the returns he is likely to achieve. Therefore, I use a method combining the variabilities inherent in all the relevant factors under consideration. The objective is to give a clear picture of the relative risk and the probable odds of coming out ahead or behind in light of uncertain foreknowledge.

A simulation of the way these factors may combine as the future unfolds is the key to extracting the maximum information from the available forecasts. In fact, the approach is very simple, using a computer to do the necessary arithmetic. To carry out the analysis, a company must follow three steps:

1. Estimate the range of values for each of the factors (for example, range of selling price and sales growth rate) and within that range the likelihood of occurrence of each value.
2. Select at random one value from the distribution of values for each factor. Then combine the values for all of the factors and compute the rate of return (or present value) from that combination. For instance, the lowest in the range of prices might be combined with the highest in the range of growth rate and other factors. (The fact that the elements are dependent should be taken into account, as we shall see later.)

3. Do this over and over again to define and evaluate the odds of the occurrence of each possible rate of return. Since there are literally millions of possible combinations of values, we need to test the likelihood that various returns on the investment will occur. This is like finding out by recording the results of a great many throws what percent of 7s or other combinations we may expect in tossing dice. The result will be a listing of the rates of return we might achieve, ranging from a loss [if the factors go against us] to whatever maximum gain is possible with the estimates that have been made.

For each of these rates we can determine the chances that it may occur. (Note that a specific return can usually be achieved through more than one combination of events. The more combinations for a given rate, the higher the chances of achieving it—as with 7s in tossing dice.) The average expectation is the average of the values of all outcomes weighted by the chances of each occurring.

We can also determine the variability of outcome values from the average. This is important since, all other factors being equal, management would presumably prefer lower variability for the same return if given the choice. This concept has already been applied to investment portfolios.

When the expected return and variability of each of a series of investments have been determined, the same techniques may be used to examine the effectiveness of various combinations of them in meeting management objectives.

Practical test

To see how this new approach works in practice, let us take the experience of a management that has already analyzed a specific investment proposal by conventional techniques. Taking the same investment schedule and the same expected values actually used, we can find what results the new method would produce and compare them with the results obtained by conventional methods. As we shall see, the new picture of risks and returns is different from the old one. Yet the differences are attributable in no way to changes in the basic data—only to the increased sensitivity of the method to management's uncertainties about the key factors.

**Investment proposal**

In this case, a medium-size industrial chemical producer is considering a $10 million extension to its processing plant. The estimated service life of the facility is ten years, the engineers expect to use 250,000 tons of processed material worth $510 per ton at an average processing cost of $435 per ton. Is this investment a good bet? In fact, what is the return that the company may expect? What are the risks? We need to make the best and fullest use of all the market research and financial analyses that have been developed, so as to give management a clear picture of this project in an uncertain world.

The key input factors management has decided to use are market size, selling prices, market growth rate, share of market (which results in physical sales volume), investment required, residual value of investment, operating costs, fixed costs, and useful life of facilities. These factors are typical of those in many company projects that must be analyzed and combined to obtain a measure of the attractiveness of a proposed capital facilities investment.

**Obtaining estimates**

How do we make the recommended type of analysis of this proposal? Our aim is to develop for each of the nine factors listed a frequency distribution or probability curve. The information we need includes the possible range of values for each factor, the average, and some idea as to the likelihood that the various possible values will be reached.

It has been my experience that for major capital proposals managements usually make a significant investment in time and funds to pinpoint information about each of the relevant factors. An objective analysis of the values to be assigned to each can, with little additional effort, yield a subjective probability distribution.

Specifically, it is necessary to probe and question each of the experts involved—to find out, for example, whether the estimated cost of production
really can be said to be exactly a certain value or whether, as is more likely, it should be estimated to lie within a certain range of values. Management usually ignores that range in its analysis. The range is relatively easy to determine; if a guess has to be made—as it often does—it is easier to guess with some accuracy a range rather than one specific value. I have found from experience that a series of meetings with management personnel to discuss such distributions are most helpful in getting at realistic answers to the a priori questions. (The term realistic answers implies all the information management does not have as well as all that it does have.)

The ranges are directly related to the degree of confidence that the estimator has in the estimate. Thus certain estimates may be known to be quite accurate. They would be represented by probability distributions stating, for instance, that there is only 1 chance in 10 that the actual value will be different from the best estimate by more than 10%. Others may have as much as 100% ranges above and below the best estimate.

Thus we treat the factor of selling price for the finished product by asking executives who are responsible for the original estimates these questions:

> Given that $510 is the expected sales price, what is the probability that the price will exceed $550?
> Is there any chance that the price will exceed $650?
> How likely is it that the price will drop below $475?

Managements must ask similar questions for all of the other factors until they can construct a curve for each. Experience shows that this is not as difficult as it sounds. Often information on the degree of variation in factors is easy to obtain. For instance, historical information on variations in the price of a commodity is readily available. Similarly, managements can estimate the variability of sales from industry sales records. Even for factors that have no history, such as operating costs for a new product, those who make the average estimates must have some idea of the degree of confidence they have in their predictions, and therefore they are usually only too glad to express their feelings. Likewise, the less confidence they have in their estimates, the greater will be the range of possible values that the variable will assume.

This last point is likely to trouble businessmen. Does it really make sense to seek estimates of variations? It cannot be emphasized too strongly that the less certainty there is in an average estimate, the
more important it is to consider the possible variation in that estimate.

Further, an estimate of the variation possible in a factor, no matter how judgmental it may be, is always better than a simple average estimate, since it includes more information about what is known and what is not known. This very lack of knowledge may distinguish one investment possibility from another, so that for rational decision making it must be taken into account.

This lack of knowledge is in itself important information about the proposed investment. To throw any information away simply because it is highly uncertain is a serious error in analysis that the new approach is designed to correct.

### Computer runs

The next step in the proposed approach is to determine the returns that will result from random combinations of the factors involved. This requires realistic restrictions, such as not allowing the total market to vary more than some reasonable amount from year to year. Of course, any suitable method of rating the return may be used at this point. In the actual case, management preferred discounted cash flow for the reasons cited earlier, so that method is followed here.

A computer can be used to carry out the trials for the simulation method in very little time and at very little expense. Thus for one trial 3,600 discounted cash flow calculations, each based on a selection of the nine input factors, were run in two minutes at a cost of $15 for computer time. The resulting rate-of-return probabilities were read out immediately and graphed. The process is shown schematically in Exhibit II.

### Data comparisons

The nine input factors described earlier fall into three categories:

1. **Market analyses**—Included are market size, market growth rate, the company's share of the market, and selling prices. For a given combination of these factors sales revenue may be determined for a particular business.

2. **Investment cost analyses**—Being tied to the kinds of service-life and operating-cost characteristics expected, these are subject to various kinds of error and uncertainty, for instance, automation progress makes service life uncertain.

3. **Operating and fixed costs**—These also are subject to uncertainty but are perhaps the easiest to estimate.

These categories are not independent, and for realistic results my approach allows the various factors to be tied together. Thus if price determines the total market, we first select from a probability distribution the price for the specific computer run and then use for the total market a probability distribution that is logically related to the price selected.

We are now ready to compare the values obtained under the new approach with those obtained by the old. This comparison is shown in Exhibit III.

### Valuable results

How do the results under the new and old approaches compare? In this case, management had been informed, on the basis of the one-best-estimate approach, that the expected return was 25.2% before taxes. When we run the new set of data through the computer program, however, we get an expected return of only 14.6% before taxes. This surprising difference results not only from the range of values under the new approach but also from the weighing of each value in the range by the chances of its occurrence.

Our new analysis thus may help management to avoid an unwise investment. In fact, the general result of carefully weighing the information and lack of information in the manner I have suggested is to indicate the true nature of seemingly satisfactory investment proposals. If this practice were followed, managements might avoid much overcapacity.

The computer program developed to carry out the simulation allows for easy insertion of new variables. But most programs do not allow for dependence relationships among the various input factors. Further, the program used here permits the choice of a value for prices from one distribution, which value determines a particular probability distribution [from among several] that will be used to determine the values for sales volume. The following scenario shows how this important technique works:

Suppose we have a wheel, as in roulette, with the numbers from 0 to 15 representing one price for the product or material, the numbers 16 to 30 representing a second price, the numbers 31 to 45 a third price, and so on. For each of these segments we would have a different range of expected market
Capital investment

volumes—for example, $150,000-$200,000 for the first, $100,000-$150,000 for the second, $75,000-$100,000 for the third. Now suppose we spin the wheel and the ball falls in 37. This means that we pick a sales volume in the $75,000-$100,000 range. If the ball goes in 11, we have a different price, and we turn to the $150,000-$200,000 range for a sales volume.

Most significant, perhaps, is the fact that the program allows management to ascertain the sensitivity of the results to each or all of the input factors. Simply by running the program with changes in the distribution of an input factor, it is possible to determine the effect of added or changed information (or lack of information). It may turn out that fairly large changes in some factors do not significantly affect the outcomes. In this case, as a matter of fact, management was particularly concerned about the difficulty in estimating market growth. Running the program with variations in this factor quickly demonstrated that for average annual growth rates from 3% to 5% there was no significant difference in the expected outcome.

In addition, let us see what the implications are of the detailed knowledge the simulation method gives us. Under the method using single expected values, management arrives only at a hoped-for expectation of 23.2% after taxes (which, as we have seen, is wrong unless there is no variability in the many input factors—a highly unlikely event).

With the proposed method, however, the uncertainties are clearly portrayed, as shown in Exhibit IV. Note the contrast with the profile obtained under the conventional approach. This concept has been used also for evaluation of product introductions, acquisition of businesses, and plant modernization.

Comparing opportunities

From a decision-making point of view one of the most significant advantages of the new method of determining rate of return is that it allows management to discriminate among measures of (1) expected return based on weighted probabilities of all possible returns, (2) variability of return, and (3) risks.

To visualize this advantage, let us take an example based on another actual case but simplified for purposes of explanation. The example involves two investments under consideration, A and B. With the investment analysis, we obtain the tabulated and plotted data in Exhibit V. We see that:

- Investment B has a higher expected return than Investment A.
- Investment B also has substantially more variability than Investment A. There is a good chance that Investment B will earn a return quite different

<table>
<thead>
<tr>
<th>Exhibit III</th>
<th>Comparison of expected values under old and new approaches</th>
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</thead>
<tbody>
<tr>
<td><strong>Market analyses</strong></td>
<td></td>
</tr>
<tr>
<td><strong>1. Market size</strong></td>
<td></td>
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<tr>
<td>Expected value (in tons)</td>
<td>250,000</td>
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<tr>
<td>Range</td>
<td>—</td>
</tr>
<tr>
<td><strong>2. Selling prices</strong></td>
<td></td>
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<tr>
<td>Expected value (in dollars/ton)</td>
<td>$510</td>
</tr>
<tr>
<td>Range</td>
<td>—</td>
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<tr>
<td><strong>3. Market growth rate</strong></td>
<td></td>
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<tr>
<td>Expected value</td>
<td>3%</td>
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<tr>
<td>Range</td>
<td>—</td>
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<td><strong>4. Eventual share of market</strong></td>
<td></td>
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<tr>
<td>Expected value</td>
<td>12%</td>
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<tr>
<td>Range</td>
<td>—</td>
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<tr>
<td><strong>Investment cost analyses</strong></td>
<td></td>
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<tr>
<td><strong>5. Total investment required</strong></td>
<td></td>
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<tr>
<td>Expected value (in $ millions)</td>
<td>$9.5</td>
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<tr>
<td>Range</td>
<td>—</td>
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<tr>
<td><strong>6. Useful life of facilities</strong></td>
<td></td>
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<tr>
<td>Expected value (in years)</td>
<td>10</td>
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<tr>
<td>Range</td>
<td>—</td>
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<tr>
<td><strong>7. Residual value (at 10 years)</strong></td>
<td></td>
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<tr>
<td>Expected value (in $ millions)</td>
<td>$4.5</td>
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<tr>
<td>Range</td>
<td>—</td>
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<tr>
<td><strong>Other costs</strong></td>
<td></td>
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<tr>
<td><strong>8. Operating costs</strong></td>
<td></td>
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<tr>
<td>Expected value (in dollars/ton)</td>
<td>$435</td>
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<tr>
<td>Range</td>
<td>—</td>
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<tr>
<td><strong>9. Fixed costs</strong></td>
<td></td>
</tr>
<tr>
<td>Expected value (in $ thousands)</td>
<td>$300</td>
</tr>
<tr>
<td>Range</td>
<td>—</td>
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</tbody>
</table>

Note: Range figures in right-hand column represent approximately 1% to 99% probabilities. That is, there is only a 1-in-100 chance that the value actually achieved will be respectively greater or less than the range.
from the expected return of 6.8%—possibly as high as 15% or as low as a loss of 5%. Investment A is not likely to vary greatly from the anticipated 5% return.

Investment B involves far more risk than does Investment A. There is virtually no chance of incurring a loss on Investment A. However, there is a chance in 10 of losing money on Investment B. If such a loss occurs, its expected size is approximately $200,000.

Clearly, the new method of evaluating investments provides management with far more information on which to base a decision. Investment decisions made only on the basis of maximum expected return are not unequivocally the best decisions.

The question management faces in selecting capital investments is first and foremost: What information is needed to clarify the key differences among various alternatives? There is agreement as to the basic factors that should be considered—markets, prices, costs, and so on. And the way the future return on the investment should be calculated, if not agreed on, is at least limited to a few methods, any of which can be consistently used in a given company. If the input variables turn out as estimated, any of the methods customarily used to rate investments should provide satisfactory (if not necessarily maximum) returns.

In actual practice, however, the conventional methods do not work out satisfactorily. Why? The reason, as we have seen earlier in this article and as every executive and economist knows, is that the estimates used in making the advance calculations are just that—estimates. More accurate estimates would be helpful, but at best the residual uncertainty can easily make a mockery of corporate hopes. Nevertheless, there is a solution. To collect realistic estimates for the key factors means to find out a great deal about them. Hence the kind of uncertainty that is involved in each estimate can be evaluated ahead of time. Using this knowledge of uncertainty, executives can maximize the value of the information for decision making.

The value of computer programs in developing clear portrayals of the uncertainty and risk surrounding alternative investments has been proved. Such programs can produce valuable information about the sensitivity of the possible outcomes to the variability of input factors and to the likelihood of achieving various possible rates of return. This information can be extremely important as a backup to management judgment. To have calculations of the odds on all possible outcomes lends some assurance to the decision makers that the available information has been used with maximum efficiency.

This simulation approach has the inherent advantage of simplicity. It requires only an extension of the input estimates (to the best of our ability) in terms of probabilities. No projection should be pinpointed unless we are certain of it.

The discipline of thinking through the uncertainties of the problem will in itself help to ensure improvement in making investment choices. For to understand uncertainty and risk is to understand
the key business problem—and the key business opportunity. Since the new approach can be applied on a continuing basis to each capital alternative as it comes up for consideration and progresses toward fruition, gradual progress may be expected in improving the estimation of the probabilities of variation.

Lastly, the courage to act boldly in the face of apparent uncertainty can be greatly bolstered by the clarity of portrayal of the risks and possible rewards. To achieve these lasting results requires only a slight effort beyond what most companies already exert in studying capital investments.

**Retrospective commentary**

When this article was published 15 years ago, there were two recurrent themes in the responses of the management community to it: (1) how the uncertainties surrounding each key element of an investment decision were to be determined, and (2) what criteria were to be used to decide to proceed with an investment once the uncertainties were quantified and displayed.

I answered the latter question in an HBR sequel, "Investment Policies That Pay Off," describing the relationships of risks and stakes to longer term investment criteria. This article, published in 1968, showed how risk analyses can provide bases for developing policies to choose among a variety of investment alternatives. Similar approaches were subsequently developed for investment fund portfolio management.

The analysis of uncertainty in describing complex decision-making situations is now an integral part of business and government. The elements of an investment decision—private or public—are subject to all the uncertainties of an unknown future. As the 1964 article showed, an estimated probability distribution paints the clearest picture of all possible outcomes. Such a description contains considerably more information than simplistic combinations of subjective best estimates of input factors. Best estimates are point estimates (there may be more than one—high, medium, low) of the value of an element of the investment analysis used for determining an outcome decision criterion, such as internal rate of return or present value of the investment.

Thus even where the conventional approach was used for the best estimate in a single-point determination for the statistically estimated expected values from a distribution of an element, the single-point approach was shown to be exceedingly misleading. In Exhibit III, a single-point best-estimate analysis gave an internal rate of return of 25.2%. And a risk analysis employing estimated frequency distributions of the elements showed that an average of possible outcomes, weighted by the relative frequency of their occurrences at 14.6%, was more realistic as well as significantly different. It presented a truer picture of the actual average expectation of
the result of this investment (if it could be repeated over and over again).

The case was thus made, and the point of this result—that risk and uncertainty were more accurately defined by a simulation of input variables—was little questioned thereafter. Managements began to adopt some form of this procedure to examine some, if not all, significant investments where doubt existed about the risk levels involved. My sequel article attempted to demonstrate that if enough investments were chosen consistently on the basis of criteria related to these kinds of risk portrayals, the overall outcomes would stabilize around the desired expected value or best estimate of the criterion.

All this now seems simple and straightforward. Earlier it was falsely thought that risk analysis was aimed at eliminating uncertainty, which was not worth doing at all since the future is so desperately uncertain. Thus in 1970 the Financial Times [of London] published an article intended to show the futility of risk analysis. It concerned a baker of geriatric biscuits who made an investment only to go bankrupt when his nursing home market precipitately disappeared with the death of its founder. The author cited as a moral, “Don’t put all your dough in one biscuit.”

It took a while for the points to diffuse through executive circles that (1) exactly such an analysis would have been just as bad, or worse, done via single-point subjective estimates, and (2) no one analytical technique could control future events, even with sensitive inputs and requirements for follow-up control to improve the odds as projected by the original risk analyses. But in the end, judgment would be required in both input estimation and decision.

I did not intend the article to be an argument in methodology but rather a cautionary note to examine the data surrounding an investment proposal in light of all the pervasive uncertainties in the world, of which business is simply one part. The years since 1964 have made it clear to me that this message should have been amplified and more emphatically insisted on in the article.

Had this point been clearer, the issue whether to take the risk and proceed with an investment might have been less troublesome. Had I been able to look with more prescience, I might have seen that the area of risk analysis would become routine in business and virtually universally adopted in public cost-benefit issues.

Cost-benefit analysis for public decisions is, of course, only a special form of investment analysis. Government issues that require decisions involving significant uncertainty are too numerous to catalog fully—energy, from both fossil and nuclear sources; chemical, drug, and food carcinogen hazards; DNA manipulation and its progeny of gene splicing.

The Three Mile Island nuclear accident brought home the fallibility of stating a risk analysis conclusion in simplistic terms. The well-known Rasmussen report on nuclear reactor safety, commissioned by the Nuclear Regulatory Commission, undertook what amounted to a risk analysis that was intended to provide a basis for investment decisions relating to future nuclear energy production. The Nuclear Regulatory Commission, in January 1979, disclaimed the risk estimates of that report; new studies to estimate risk are now underway. But there is also a school of thought saying we face too many risks each day to worry about one more.

A commonly stated estimate of the risk of a major nuclear power plant accident is 1 chance in 1,000,000 years. In the 1964 article, I portrayed the image of risk with a chart of the throws of two dice that would be required to give various outcomes—from two 1s to two 6s, each of these having a 1-in-36 chance of occurring. There should be no problem in visualizing or testing the meaning and the chances of any of the events pictured by these dice. And, although 1 in 1,000,000 is somehow presented as “mind boggling” compared with 1 in 36, and so unlikely to occur as to be beyond our ken, I suggest that it is just as simply visualized.

We simply need to use eight dice at once. If we chart all the possible outcomes for eight dice, as we did for the two, we find that the sum of 8 (or 48) can occur just one way—via all 1s (or all 6s). The odds of this occurring are roughly 1 in 1,680,000. Thus the visualization of such odds and more important, the lesson we must learn about risk—which incidents like Three Mile Island should teach us—is that what can happen will happen if we just keep at it long enough. Any of us can simulate a statistical picture of the estimated risks or even the complexities of the Rasmussen analysis with enough patience and enough dice (or a computer).

Incidentally, to make the eight dice act more like the odds of 1 in 1,000,000, simply mark any two “non-1” sides with a felt pen and count them as 1s if they turn up; the odds of getting all 1s become a little less than 1 in 1,100,000. And the chances of human error can be included by similarly marking other dice in the set. The difficulty is not in constructing such a simulation to portray the odds but in determining events that may lead to these odds and estimating the frequencies of their occurrence.
Risk analysis has become one with public policy. Without it, any important choice that leads to uncertain outcomes is uninformed; with it, properly applied and understood, the decision maker—business executive, government administrator, scientist, legislator—is better able to decide why one course of action might be more desirable than another.

The fear of risk-taking

To try to eliminate risk in business enterprise is futile. Risk is inherent in the commitment of present resources to future expectations. Indeed, economic progress can be defined as the ability to take greater risks. The attempt to eliminate risks, even the attempt to minimize them, can only make them irrational and unbearable. It can only result in the greatest risk of all: rigidity.

The main goal of a management science must be to enable business to take the right risk. Indeed, it must be to enable business to take greater risks—by providing knowledge and understanding of alternative risks and alternative expectations: by identifying the resources and efforts needed for desired results; by mobilizing energies for contribution; and by measuring results against expectations, thereby providing means for early correction of wrong or inadequate decisions.

All this may sound like mere quibbling over terms. Yet the terminology of risk minimization does induce a decided animus against risk-taking and risk-making—that is, against business enterprise—in the literature of the management sciences. Much of it echoes the tone of the technocrats of a generation ago.

For it wants to subordinate business to technique, and it seems to see economic activity as a sphere of physical determination rather than as an affirmation and exercise of responsible freedom and decision.

This is worse than being wrong. This is lack of respect for one's subject matter—the one thing no science can afford and no scientist can survive. Even the best and most serious work of good and serious people—and there is no lack of them in the management sciences—is bound to be vitiated by it.

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